

Game Theory 3:

Dynamic Games & Logic

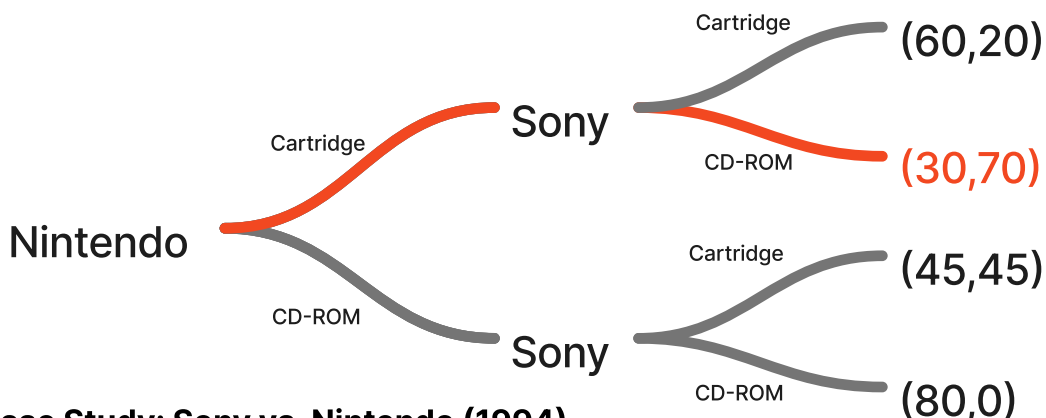


GAME OF GAINS

The Game Tree (Extensive Form)

Core Concepts of Extensive Form

- **Extensive Form:** A map of a game represented as a tree, showing the Sequence of Play, decision nodes, and available information.
- **Node:** A specific point in the tree where a player must choose an action.
- **Root Node:** The initial point of the game where the first player makes a move.
- **Terminal Node:** The final point of a branch where the game ends and payoffs are distributed.
- **Branches:** Lines connecting nodes that represent the specific actions or choices available to a player.
- **Information Sets:** Dotted lines or circles around nodes indicating that a player cannot distinguish which specific node they are at when making a move.



Case Study: Sony vs. Nintendo (1994)

- **Context:** In the early 90s, Nintendo dominated the market. They had to decide between the high-capacity, piracy-prone CD-ROM or their traditional, high-control Cartridges. Sony, a new entrant, waited to see Nintendo's choice before finalizing the PlayStation's specs.
- **Root Node:** Nintendo moves first, choosing the Cartridge branch. They prioritize manufacturing royalties and hardware security, aiming for a 60 payoff.
- **Decision Node:** Sony acts as the second mover. Because this is a dynamic game, Sony observes Nintendo's hardware commitment before making their move.
- **The Logic:** Sony evaluates the branches. If they match Nintendo with cartridges, their payoff is low (20) against an established incumbent. If they disrupt with CD-ROM technology, their payoff jumps to 70 as third-party developers flee Nintendo's high costs.
- **The Outcome:** Sony selects the CD-ROM path, leading to a terminal node where they earn 70 (market leadership) and Nintendo's payoff drops to 30. Sony's success came from mapping the branches and choosing the action that exploited Nintendo's "locked-in" first move.

Solving for SPNE

Equilibrium and Rationality

- **Subgame Perfect Nash Equilibrium (SPNE):** An equilibrium that represents a Nash Equilibrium in every part (subgame) of the larger game, eliminating "non-credible threats."
- **Subgame:** A smaller piece of the total game tree that starts at a single node and contains all its succeeding branches.
- **Backward Induction:** The process of reasoning backward from the end of a situation to determine the optimal sequence of actions.
- **Credibility:** A central tenet of SPNE; it ensures that players only consider threats or promises that would actually be in the opponent's best interest to carry out.

Methodology and Pruning

- **Solving the Tree:** Analysis begins at the terminal nodes (the end) and moves toward the root.
- **Optimization:** At each final decision point, the analyst identifies the highest payoff for the acting player.
- **Path Selection:** [visual] Sub-optimal branches are "pruned" or grayed out to show the rational path to the equilibrium.
- **Final Result:** The remaining "unpruned" path from the root to the terminal node constitutes the SPNE.

Case Study: Sky TV vs. BSB

- **Context:** Two satellite television providers engaged in a ruinous price war for UK market dominance.
- **The Conflict:** Both firms faced massive losses due to high bidding for programming rights and hardware subsidies.
- **Rational Induction:** By reasoning backward, these firms realized continued competition would lead to mutual bankruptcy, making a merger the only rational subgame perfect outcome.
- **Resolution:** The 1990 merger into BSkyB saved both entities by removing the sub-optimal "Continue Fighting" branch from their strategic tree.

Sequence & Commitment

The Power of Commitment

- **Commitment:** An irreversible action that changes the payoffs of the game for all players.
- **Strategy Set:** The total range of possible moves available to a player throughout the game.
- **Strategic Limiting:** Paradoxically, a player can often achieve a better outcome by strictly limiting their own future options.
- **Signaling:** A commitment serves as a powerful signal to opponents, forcing them to recalculate their own optimal response based on your new, restricted reality.

Requirements for Effective Commitment

- **Irreversibility:** The action must be difficult or impossible to undo (e.g., "burning bridges").
- **Visibility:** The opponent must be aware that the commitment has been made.
- **Clarity:** The consequences of the commitment must be easily understood by all participants in the game.

Case Study: Hernan Cortes (1519)

- **The Scenario:** Upon landing in Mexico, Cortes faced a superior force and a crew tempted to retreat to Cuba.
- **The Action:** Cortes ordered the literal burning of his own fleet.
- **Strategic Impact:** By destroying his ships, Cortes removed "Retreat" from his own strategy set.
- **Psychological Shift:** This signaled to the enemy that he was committed to a "Fight to the Death" strategy, forcing a change in the enemy's rational response.
- **Outcome:** Because the "Retreat" branch no longer existed, the Spanish forces had no choice but to advance, and the Aztec defenders had to face an opponent with zero incentive to flee.