

Game Theory 2:

Equilibria & Advanced Static Games



GAME OF GAINS

Stability vs. Optimality

Nash Equilibrium (NE)

- **The Best Response:** NE occurs when every player's strategy is their "best response" to the strategies chosen by everyone else. No one has an incentive to deviate because any single move would result in a lower or equal payoff.
- **Self-Enforcing Nature:** Unlike a contract that requires legal enforcement, a Nash Equilibrium is self-enforcing. The logic of the game itself keeps players in place.
- **Non-Uniqueness:** A game can have multiple Nash Equilibria (e.g., driving on the left vs. the right side of the road). Neither is inherently "better" until a choice is made.
- **The Logic of Stability:** It represents a state of "strategic rest." Even if the outcome is bad for everyone (like an arms race), it remains stable because no one can safely stop first.

Pareto Efficiency

- **Resource Maximization:** An outcome is Pareto Efficient if there is no "free lunch" left—you cannot help one person without hurting another.
- **The Pareto Frontier:** This is the set of all outcomes that are efficient. In a graph, it represents the outer edge of possible utility.
- **Contrast with Nash:** A Nash Equilibrium is about individual rationality, while Pareto Efficiency is about social/systemic efficiency. They often conflict (e.g., The Prisoner's Dilemma, where the NE is Pareto Inefficient).

Case Study: QWERTY and Path Dependency

- **Historical Constraint:** Designed in 1873 to slow down typists and prevent mechanical jams. As typing speed increased, the layout became an obstacle.
- **Network Effects:** The value of QWERTY isn't in its design, but in the fact that everyone else uses it. Employers hire QWERTY typists; schools teach QWERTY.
- **The Trap:** Even though the Dvorak Simplified Keyboard is statistically faster and more ergonomic (a Pareto improvement), the cost for any one person to switch is too high.
- **Path Dependency:** This illustrates how early, accidental choices can lock a system into a sub-optimal Nash Equilibrium for decades.

Coordination & Randomness

Mixed Strategies

- **The Indifference Principle:** In a mixed strategy, you choose your probabilities (e.g., 60% Left, 40% Right) so that your opponent earns the exact same expected payoff regardless of what they do.
- **Removing Exploitation:** If you are 100% predictable, a rational opponent will always defeat you. Randomization "blinds" the opponent's ability to counter-strat.
- **Equilibrium in Probability:** Every finite game has at least one Nash Equilibrium, though it may require players to use mixed strategies rather than "pure" (single-choice) moves.

Coordination Games & Focal Points

- **The Alignment Problem:** In these games, players don't compete; they struggle to agree. The challenge is "Mutual Consistency."
- **Schelling Points (Focal Points):** These are solutions people tend to choose by default in the absence of communication.
- **Types of Coordination:**
 - **Pure Coordination:** Payoffs are identical (e.g., which side of the road to drive on).
 - **Battle of the Sexes:** Coordination is desired, but players prefer different versions of the outcome.

Case Study: Penalty Kicks (The Kicker vs. The Goalie)

- **Zero-Sum Conflict:** The kicker wants a goal; the goalie wants a save. It is a game of pure anticipation.
- **Statistical Evidence:** Studies of professional soccer show that elite players follow mixed-strategy Nash Equilibrium probabilities almost perfectly.
- **The "Indifference" Application:** If a kicker is slightly better at shooting right, they must still shoot left occasionally. If they didn't, the goalie would always dive right, making the kicker's "best" side their "worst" outcome.